Factors Influencing Achievement in Mathematics Literacy

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The purpose of this study is to identify distinct factors from a set of variables intended to predict student achievement in math literacy. According to Mullis, et al. (1998), the items on the Third International Mathematics and Science Study (TIMSS) that address mathematics literacy are intended to measure how well students can use their knowledge of mathematics to solve real-world problems. Because a primary goal of mathematics education is to facilitate the transfer of mathematics knowledge from the classroom to students' lives outside the classroom, the identification of factors that correlate with that transfer is a worthwhile pursuit. While we may not be able to say that those factors *cause* transfer, we can show that when those factors are present then transfer is likely to occur. Furthermore, we can attempt to ensure the presence of those factors through the use of motivational and instructional strategies.

Participants

The TIMSS testing included more than half a million students in five different grade levels across more than 40 countries (Mullis et al., 1998). The subset of data used in the current study (n = 582) was randomly selected from U.S. students in their final year of secondary school.

Factors

Several variables were excluded from the analysis because they did not logically or adequately represent underlying dimensions. For example, variables related to activities in which the student engaged outside of school (with the exception of studying) were considered too diverse to represent a single factor. Similarly, types of mathematics learning activities were too varied to represent a single factor. Variables related to socio-economic status were considered, but only two (highest education level of mother and of father) were available. Two variables are generally considered insufficient to form a factor.

The remaining variables were hypothesized to form three distinct factors. The first factor is "Time spent studying outside of school" and consists of three variables:

- csbmday8 MAT\OUTSIDE SCHL\STUDYING MATH
- csbsday9 SCI\OUTSIDE SCHL\STUDYING SCIENCE
- csbgday0 GEN\OUTSIDE SCHL\STUDYING OTHER SUBJ

These variables range from 1="No time" to 5="More than 5 hours. This factor is an indicator of a student's "work ethic." Several studies have demonstrated the importance of academic learning time to learning achievement (Berliner, 1991; Brown & Saks, 1986; Squires, Huitt, & Segars, 1983). Therefore we expect that in general those students who spent more time studying outside of school would have higher math literacy scores than those who spent less time.

The second factor is "Attitude toward mathematics" and consists of six variables:

- csbmenjyrev THINK\ENJOY LEARNING MATH-rev
- csbmborerev THINK\MATH IS BORING-rev
- csbmeasyrev THINK\MATH IS AN EASY SUBJECT-rev
- csbmliferev THINK\MATH IS IMPORTANT IN LIFE-rev
- csbmworkrev THINK\LIKE JOB INVOLVING MATH-rev
- csbmlikerev LIKE MATHEMATICS-rev

This factor is an indicator of a student's interest in mathematics and perception of its relevance to the student's life, both of which are important components in learning as described by expectancy-value theory and the ARCS model of motivation (Keller, 1983, 1987). Negatively worded items were previously reversed in this data set, and all items in the second and third factors have been reversed again so that their Likert scales range from 1="Strongly disagree" to 4="Strongly agree." Having the values progress in this direction makes them easier to interpret.

The third factor is "Attribution of success in mathematics" and consists of four variables:

- csbmdow1rev MAT\DO WELL\NATURAL TALENT-rev
- csbmdow2rev MAT\DO WELL\GOOD LUCK-rev
- csbmdow3rev MAT\DO WELL\HARD WORK STUDYING-rev
- csbmdow4rev MAT\DO WELL\MEMORIZE NOTES-rev

This factor is an indicator of what a student thinks is required to do well in mathematics. According to attribution theory (Weiner, 1972), a student's expectations, achievement, and affective reactions are influenced by attributional conclusions. Therefore we may find a high correlation between these variables and those related to attitude and motivation.

Principal Axis Factor (PAF) Analyses

Descriptive statistics indicate sufficient variability as described by the standard deviations for the items. The item related to studying science outside of school is highly skewed, and the item related to doing well being the result of hard work is highly kurtotic; otherwise the data appear normal. The correlation matrix has pockets of high and low correlations, suggesting multidimensionality; many of the correlations are significant at 0.01, which may be attributed in part to the large sample size.

Some of the extracted communalities are low, in particular those related to the "attribution" factor, indicating that the three-factor structure explains little of the variance for those items. Therefore they are not expected to load heavily in the factor matrix. The eigenvalues for the three factors range from 3.785 to 1.384 and do not indicate unidimensionality. Overall the three factors account for 55.3% of the total variance explained. The scree plot also confirms that it is reasonable to model three factors.

The rotated factor matrix (Varimax) shows loadings that are mostly consistent with the factors hypothesized at the start of the analysis. The items were sequenced during selection so that they would group into factors in the factor matrix. The first three items comprise the factor "Time spent studying outside of school," which ends up being factor 2 in the matrix. The next six items comprise the factor "Attitude toward mathematics," which is factor 1 in the matrix; however, the final item in that list—"Think math is important in life"—loads almost as heavily on factor 2 (.280 and .204 respectively). These low loadings indicate that the variable is not well represented in the factor space. The last four items were hypothesized to comprise the factor "Attribution of success in mathematics," which is factor 3 in the matrix; however, the final item—" Doing well being the result of hard work," which was found to be highly kurtotic—loads most heavily on factor 2 instead (.161 and .394 respectively). This may be because students associate working hard on math with studying outside of school.

While the factor loading values differ among the rotated factor matrix (Varimax), the pattern matrix (Promax), and the structure matrix (Promax), the loading patterns (i.e. the factor on which each item loads most heavily) are the same. Additionally, the factor correlation matrix for the oblique (Promax) rotation shows low correlations between factors. Therefore the orthogonal (Varimax) rotation is preferred for these data.

PAF Varimax Calculations

Reproduced correlation between the first two variables.

	A			A'		R_{repro}	
var1	0.237	0.687	-0.052	0.237	0.151	0.531	0.434
var2	0.151	0.573	-0.092	0.687	0.573	0.434	0.360
				-0.052	-0.092		

Residual of the original and reproduced correlations.

	R_{obs}		R_{repro}		R_{resid}	
	var1	var2	var1	var2	var1	var2
var1	1	0.512	0.531	0.434		0.078
var2	0.512	1	0.434	0.360	0.078	

"A" is the matrix of factor loadings for the variables and "A`" is the inverse. They are multiplied to obtain R_{repro}. The difference between R_{obs} and R_{repro} is the residual, R_{resid}. These figures match those in the SPSS printout.

Communality for Variable 1

MAT\OUTSIDE SCHL\STUDYING MATH

	Loadings			Squared Loadings			SSL
	1	2	3	1^2	2^2	3^2	
Unrotated	0.428	0.588	0.044	0.183	0.346	0.002	0.531
Rotated	0.237	0.687	-0.052	0.056	0.472	0.003	0.531

The communality (SSL or sum of squared loadings) for the variable is the same before and after rotation. Rotation is intended to simplify the structure and make interpretation easier, but it does not change the amount of variance in the item that is accounted for by the factors.

Interpretation of Residual Correlation Matrix

The factor solution does a reasonable job of reproducing the correlation matrix, with 11 (14%) nonredundant residuals with absolute values greater than 0.05. The problematic variable "Doing well being the result of hard work," which was found to be highly kurtotic and which loaded most heavily on factor 2 instead of factor 3 as expected, is not well reproduced by the factor solution, with some of the highest residuals. It is tempting to question the inclusion of this variable in the model, but theoretically it should be reasonable to consider it part of the "attribution" factor.

Maximum Likelihood Analysis

The communalities for the ML oblique (Promax) solution are different than those for the PAF oblique (Promax) solution. This is not surprising because the methods have different maximizing criteria. However, the differences may have been exacerbated by the fact that communality estimates greater than 1 were encountered during iterations for the ML solution.

Therefore the results must be interpreted with caution. One possible reason is that the number of factors extracted is wrong, but conducting the analysis again without specifying the number of factors still yielded the same problem. Nevertheless, the eigenvalues are the same for both solutions.

The χ^2 goodness-of-fit test indicates a statistically significant difference between R_{obs} and R_{repro}. The ML solution is sensitive to large sample sizes (n=551 in this case) and non-normality, which was suspected based on some skewness and kurtosis seen in the descriptive statistics. An alternative to overcome the limitations of the raw χ^2 is to use the measure χ^2 / df ≈ 3. In this case, 140.799 / 42 = 3.35.

The ML solution results in a slightly better residual correlation matrix than the PAF solution, with 9 (11%) nonredundant residuals with absolute values greater than 0.05. The pattern and structure matrices for the two solutions are similar, and the ML factor correlation matrix also suggests that an orthogonal rotation is preferred.

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